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Adaptation of the Thornthwaite scheme for estimating daily reference evapotranspiration

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Abstract

A photoperiodic effective daily temperature is proposed as an alternative version of the Thornthwaite scheme for estimating daily evapotranspiration rates. Extensive lysimetric data sets from two contrasting climatic regions were used to test the proposed version. The well-known gross underprediction of the original Thornthwaite approach for arid climates became a slight overprediction under the new version, but it was equivalent to that obtained with the well-recommended Penman–Monteith FAO-56 (PM-56) parameterization scheme. The PM-56 schemes performed well for both environments.

Keywords: Reference ET; Thornthwaite approach; Empirical method; Penman–Monteith

1. Introduction

The Food and Agriculture Organization (FAO) released the Irrigation and Drainage Paper 56 (Allen et al., 1998) recommending that the reference evapotranspiration (ET_0) be computed solely based on the parameterizations proposed by Allen et al. (1989) for the Penman–Monteith equation (Monteith, 1965). Even though such proposal has been tested positively in many climates, the need for many input variables, seldom available in remote areas of developing regions, limits its widespread use. An alternative is the use of regional weather stations providing the necessary inputs. However, such rarely available stations require qualified personnel for operation and maintenance of the very sensitive instruments. Instrument calibration is not a common practice due to the lack of standards, even in research centers. Consequently, large measurements errors are possible as the instruments age.

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Empirical methods are the next alternative and, recently, Alexandris and Kerkides (2003) have used surface polynomial regression analysis using hourly solar radiation, air temperature, and relative humidity to estimate ET_0. A much simple alternative is the Thornthwaite scheme since it requires only temperature as input data. However, the Thornthwaite approach has been found to underestimate under arid conditions (Pelton et al., 1960; Stanhill, 1961; Pruitt, 1964; Pruitt and Doorenbos, 1977; Hashemi and Habibian, 1979; Malek, 1987) and to overestimate in the equatorial humid climate of the Amazon region (Camargo et al., 1999). Up to now, the only adjustment of the Thornthwaite scheme was proposed by Camargo et al. (1999) using an "effective" temperature instead of the original average temperature. Their proposal was based on monthly averages and totals, and it was tested against the Penman–Monteith FAO-56 ET_0 estimatives.

The objective here is to test such modification of the Thornthwaite scheme against daily lysimetric ET_0 measurements on two contrasting environments, namely the dry climate of Davis, CA, and the humid summer and the dry fall–winter of Piracicaba, SP, Brazil. A further correction based on the daily photoperiod is here proposed for the daily effective temperature.

2. Material and methods

The potential evapotranspiration (or reference evapotranspiration, ET_M, mm per month) for a standard month of 30 days, each day with 12 h of photoperiod, was computed as a function of the month average temperature (T, °C) by the scheme proposed by Thornthwaite (1948) as

$$ET_M = 16\left(10\frac{T}{I}\right)^a, \quad 0\,^\circ\mathrm{C} \leq T \leq 26\,^\circ\mathrm{C} \tag{1}$$

where I is a thermal index imposed by the local normal climatic temperature regime (T_n, °C) and the exponent a is a function of I, both computed by

$$I = \sum_{n=1}^{12}(0.2T_n)^{1.514}, \quad T_n > 0\,^\circ\mathrm{C} \tag{2}$$

$$a = 6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 1.7912 \times 10^{-2} I + 0.49239 \tag{3}$$

For temperature above 26 °C, instead of the original table of Thornthwaite (1948), Willmott et al. (1985) represented ET_M by the following equation:

$$ET_M = -415.85 + 32.24T - 0.43T^2, \quad T > 26\,^\circ\mathrm{C} \tag{4}$$

In order to convert the estimates from a standard monthly (ET_M, mm per month) to a daily time scale (ET_D, mm per day) the following correction factor (C) was used:

$$C = \frac{N}{360} \tag{5}$$

where N is the photoperiod (h) for a given day.

Camargo et al. (1999) found that the performance of the Thornthwaite approach in a monthly time scale improved if an "effective" temperature (T_{ef}) is used instead of the recommended average temperature ($T_{avg} = 0.5(T_{max} + T_{min})$). The effective temperature was computed empirically as a function of the average temperature and of the daily amplitude ($A = T_{max} - T_{min}$), as

$$T_{ef} = k(T_{avg} + A) = \tfrac{1}{2}k(3T_{max} - T_{min}) \quad (6)$$

with $k = 0.72$ as the statistically best value for estimating ET_M. Different values were tested here with $k = 0.69$ giving the best estimatives for ET_0.

However, any 2 days with the same T_{ef} but with very different photoperiods (N) are likely to have different evapotranspiration rates. It is proposed here to correct T_{ef} with the day–night ratio $N/24 - N$, or

$$T^*_{ef} = T_{ef}\frac{N}{24 - N} \quad (7)$$

with the following restrictions: $T_{avg} \leq T^*_{ef} \leq T_{max}$.

Daily reference evapotranspiration was obtained from 1960 to 1963 at University of California, Davis, CA, USA (38°32′N; 121°46′W; 19 m a.s.l.). The regional climate is classified as arid. A weighing lysimeter located near the center of a 146 m × 355 m field of perennial ryegrass mowed to keep an average plant height of 0.1 m, and irrigated weekly gave the daily ET (Pruitt, 1964). Days with strong advection as well as those with restricted ET were discarded based on the $0.5 < ET/R_n^+ < 0.9$ criteria, where R_n^+ is the summation of the positive values of measured net radiation. The screening process resulted in 306 days spanning throughout the year with lysimetric measurements ranging from 0.43 to 7.49 mm per day. The weather elements had the following ranges: $4.4\,°C \leq T_{min} \leq 15.6\,°C$; $13.3\,°C \leq T_{max} \leq 37.2\,°C$; 43 km per day ≤ wind run ≤ 498 km per day.

Another data set came from measurements at Piracicaba, SP, Brazil (22°47′S; 47°30′W; 546 m a.s.l.) during 1996. The climate of the region is tropical humid with a dry winter. ET was obtained from a weighing lysimeter located about 80 m from the leading edge of the most prevailing wind direction in a 35 m × 90 m field of *Paspalum notatum* L. grass. Soil moisture was kept near field capacity by frequent irrigations during the dry season. The grass was mowed close to the 0.12 m of a reference surface defined by Allen et al. (1998). The same ET/R_n^+ criteria was used to screen the data set. Rainy days were also discarded resulting in 127 days from January to May and August to December. The weather elements had the following ranges: $5.6\,°C \leq T_{min} \leq 21.9\,°C$; $21.5\,°C \leq T_{max} \leq 33.5\,°C$; 63 km per day ≤ wind run ≤ 311 km per day.

Following Allen et al. (1989), a standard regression analysis ($Y = bX$) was performed to test the goodness of the fit between estimated (X) versus measured ET (Y) when the Y-intercept did not differ statistically from zero. The root mean square error (RMSE, ±mm per day) and coefficient of determination (r^2) statistics were also used.

3. Results and discussion

To visualize the improvement obtained with the new proposal for the Thornthwaite scheme a comparison with the original approach is presented. As expected, for the arid

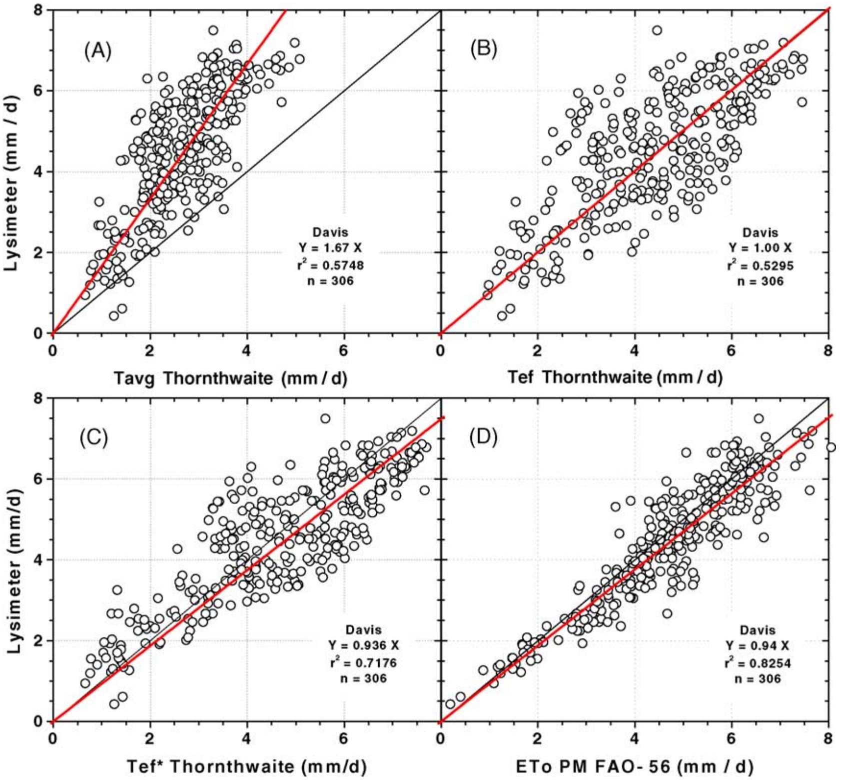

Fig. 1. Comparisons of daily reference evapotranspiration estimated by three versions of the Thornthwaite method ((A) T_{avg}, (B) T_{ef} and (C) T^*_{ef}) and the Penman–Monteith FAO-56 (D) with the measurements of a weighing lysimeter in Davis, CA.

conditions of Davis, CA, the degree of underestimation using T_{avg} as input was very large (Lys $=$ 1.67 Thorn; RMSE $= \pm 1.04$ mm per day; $r^2 = 0.5748$; $n = 306$), shown in Fig. 1A, confirmed previous report from Pruitt and Doorenbos (1977). However, when an "effective" daily temperature (T_{ef}) was used instead of the original daily average temperature, the estimates improved substantially with most of points spreading around the line of perfect fit (Lys $=$ 1.00 Thorn; RMSE $= \pm 1.13$ mm per day; $r^2 = 0.5295$; $n = 306$), as can be seen in Fig. 1B. Taking into account the different photoperiods during the year, the corrected effective temperature (T^*_{ef}) gave a slight overprediction but with less spread of the points and with a decrease in the root mean square error (Lys $=$ 0.936 Thorn; RMSE $= \pm 0.90$ mm per day; $r^2 = 0.7176$; $n = 306$).

For the sake of comparison, the Thornthwaite scheme with T^*_{ef} gave an almost identical mean relationship as that obtained with the same data set using the Penman–Monteith FAO-56 estimates (Lys $=$ 0.94 PM-56; RMSE $= \pm 0.64$ mm per day; $r^2 = 0.8254$; $n = 306$), as seen in Fig. 1D.

The evapotranspiration accumulated along the 306 days of data amounted to 1376,45 mm in the lysimeter with an average of 4.50 mm per day (0.43–7.49 mm per day). The

Thornthwaite estimates were: 807.85 mm using T_{avg} (mean of 2.64; 0.66–5.08 mm per day); 1328.61 mm with T_{ef} (4.34; 0.96–7.47 mm per day) and 1424.41 mm with T_{ef}^* (4.65; 0.66–7.69 mm per day). The Penman–Monteith FAO-56 gave a total of 1458.3 mm with a daily average of 4.77 mm (0.19–7.66 mm per day). Taking the lysimeter as reference, the Thornthwaite estimates represented about 59% with T_{avg}, 97% with T_{ef}, 103% with T_{ef}^*, and the PM-56 gave 106%.

For the data set from Piracicaba, SP, Brazil, the original Thornthwaite scheme (T_{avg}) resulted also in underprediction of 25%, on average, or Lys = 1.248 Thorn; RMSE = ±1.22 mm per day; $r^2 = 0.3310$; $n = 127$ (Fig. 2A). Using the "effective" daily temperature (T_{ef}) resulted in a substantial improvement of the estimates with the points spreading around the 1:1 line, giving the following statistics: Lys = 1.02 Thorn; RMSE = ±0.91 mm per day; $r^2 = 0.4539$; $n = 127$ (Fig. 2B). When the photoperiodic effective temperature (T_{ef}^*) was used it resulted in almost identical overprediction observed for Davis, CA, i.e., Lys = 0.956 Thorn; RMSE = ±0.87 mm per day; $r^2 = 0.6044$; $n = 127$ (Fig. 2C). The Penman–Monteith FAO-56 gave the following relationships: Lys = 0.991 PM-56; RMSE = ±0.50 mm per day; $r^2 = 0.8158$; $n = 127$ (Fig. 2D).

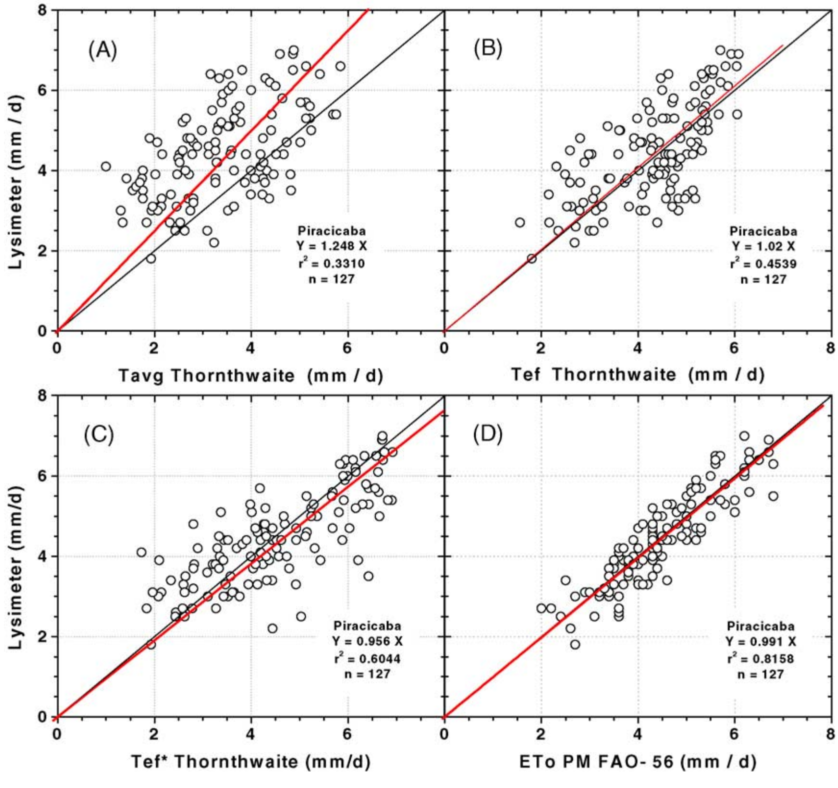

Fig. 2. Comparisons of daily reference evapotranspiration estimated by three versions of the Thornthwaite method ((A) T_{avg}, (B) T_{ef} and (C) T_{ef}^*) and the Penman–Monteith FAO-56 (D) with the measurements of a weighing lysimeter in Piracicaba, SP, Brazil.

The accumulated evapotranspiration during the 127 days resulted in 564.1 mm measured with the lysimeter with a daily mean of 4.44 mm (1.77–7.00 mm per day), while the Thornthwaite scheme with T_{avg} gave 428.0 mm (3.37; 1.00–5.84 mm per day), 546.4 mm with T_{ef} (4.30; 1.56–6.08 mm per day), and 572.3 mm (4.51; 1.73–6.92 mm per day) with T^*_{ef}. The Penman–Monteith FAO-56 totaled 569.5 mm during the same period with a daily average of 4.48 mm (2.03–6.78 mm per day). The estimates with T_{avg} represented about 76% of the total measured with the lysimeter, while T_{ef} gave 97% and T^*_{ef} estimated 102% of it. The Penman–Monteith FAO-56 gave 101% of the lysimeter total.

A detailed analysis of the ratio lysimeter/estimated T^*_{ef} did not show any significant relationship either with T_{max}, T_{min}, or wind run for both locations. In other words, no single weather element was responsible for the scatter of points (mismatch) displayed in the figures.

4. Conclusions

With data from two contrasting climatic conditions, the proposed version of the Thornthwaite scheme for estimating daily reference evapotranspiration taking as input the normal climatic temperature for determining the *I* and *a* thermal indices, and the daily effective temperature had a performance almost identical to that of the more robust and highly recommended Penman–Monteith FAO-56 model (PM-56), but with more spread of the points. In terms of totals for the whole period of measurements and overall averages, there was not much difference between the estimates with the daily effective temperature and that with the photoperiodic effective temperature. The PM-56 performed well for both environments.

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